Decidable Logics Combining Heap Structures and Data

Xiaokang Qiu[1]

P. Madhusudan[1]   Gennaro Parlato[2]

[1]University of Illinois at Urbana-Champaign
[2]LIAFA, CNRS and University of Paris

POPL'11(Principles of Programming Languages)

SMT Solvers: Engines of logical reasoning

SMT solvers

- check satisfiability in particular theories (usually QF)
- engines of proof that serve many programming verification and analysis techniques

Myriad applications

- Test input generation
- Verifying compilers
- Computing abstractions
- Invariant generation
- Floyd-Hoare style deductive verification

Theories Supported by SMT Solvers

- Equality over uninterpreted functions and predicate symbols
- Real and integer arithmetic
- Bit-vectors
- Arrays
- Tuple/Record/Enumeration types and algebraic data-types,…

- **Nelson-Oppen**: Combining Quantifier-free theories

- Heap structures + Data?

Motivation

Reasoning with Heap structures + Data

- E.g., Verification conditions for verifying binary-search tree search

  If the key *k* is below *curr*, then in the next iteration of the loop, when curr is updated, *k* will still be below *curr*.

Previous work

- HAVOC (Lahiri & Qadeer, POPL'08)
- CSL (Bouajjani et al., CONCUR'09)

Challenge

Unbounded size of heap + Quantification

Modeling Heaps

- Heaps = Graphs + Data
  - **heap locations:** nodes of the graph
  - **field pointer *f* from *n* to *n'*** : an edge from *n* to *n'* labeled *f*
  - **data field:** function $d : Loc \rightarrow Data$

- Decidable Logic for Heaps =

  Combination of

  A Decidable Logic on Graphs

  \+

  Decidable Quantifier-free Data-logic (e.g. QF-integers)

Decidable Logic on Graphs

Monadic Second-Order (MSO) logic on tree-like graphs

Tree-like graphs:

- a skeleton tree
- precise set of nodes: unary predicate in MSO
- edges: binary relations in MSO

Quite powerful: All bounded tree-width graphs;

most recursive data-structures of interest

Recursive data-structures

A regular class of skeleton trees + unary predicates + binary relations

A class of graphs defined by

- Regular class of trees: A MSO formula ψ_{Tr}
- Edge relations E_a: given as an MSO formula $E_a(x,y)$
- Vertex labels: given as an MSO formula $L_b(x)$

DECIDABLE MSO (intepreting on trees; tree automata)

Similar to graph types (Klarlund & Schwartzbach, POPL'93)

Trees with all nodes pointing to root:

```
type Tree = {
   data left,right:Tree;
   pointer root:Tree[root<(left+right)*>this &
                       empty(root^Tree.left union
                       root^Tree.right)];
}
```

Doubly-linked list:

```
type Node = {
   bool value;
   data next:Node;
   pointer prev:Node[this^Node.next={prev}];
}
```

STRAND Logic

- A new logic *STRAND*

  defined over a class of recursive data-structures $\mathcal{R}$

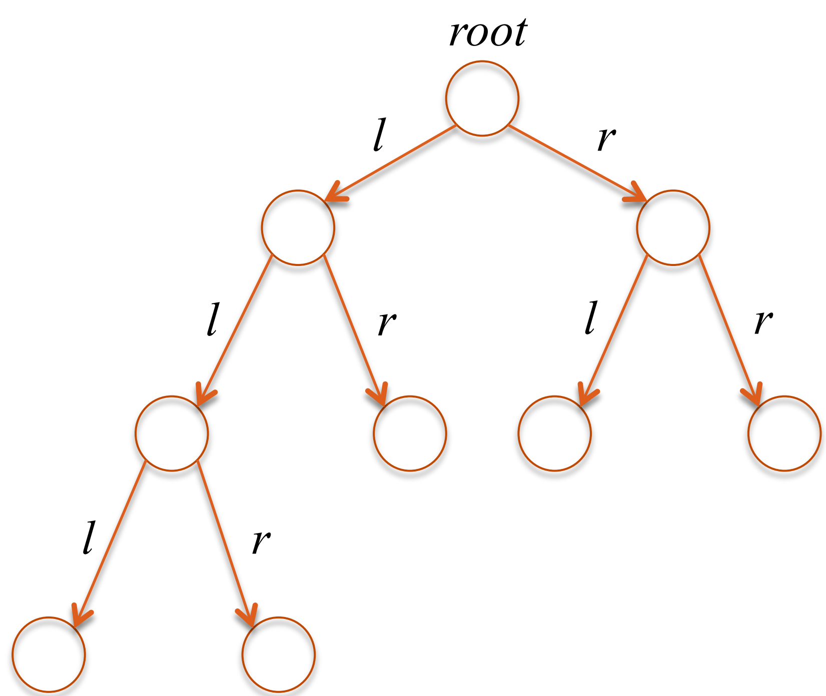
$$\exists \vec{x}\, \forall \vec{y}\, .\, \varphi(\vec{x}, \vec{y})$$

  - where φ is a MSO formula enriched by data-field functions *data*, but where the data-constraints are only allowed to refer to $\vec{x}$ and $\vec{y}$.

  - **Example:** A binary tree, with all data values at leaves being 1

$$\forall y.(\underbrace{\neg \exists z.\ (E_l(y,z) \wedge E_r(y,z)) \Rightarrow (\ data(y) = 1\ )}_{\varphi})$$

Example: Binary Search Tree



$$\psi_{bst} \equiv \forall y_1, y_2.\ (\ (\ leftsubtree\ (y_1, y_2) \Rightarrow data(y_2) \leq data(y_1))$$

$$\wedge\ (\ rightsubtree\ (y_1, y_2) \Rightarrow data(y_2) > data(y_1))\ )$$

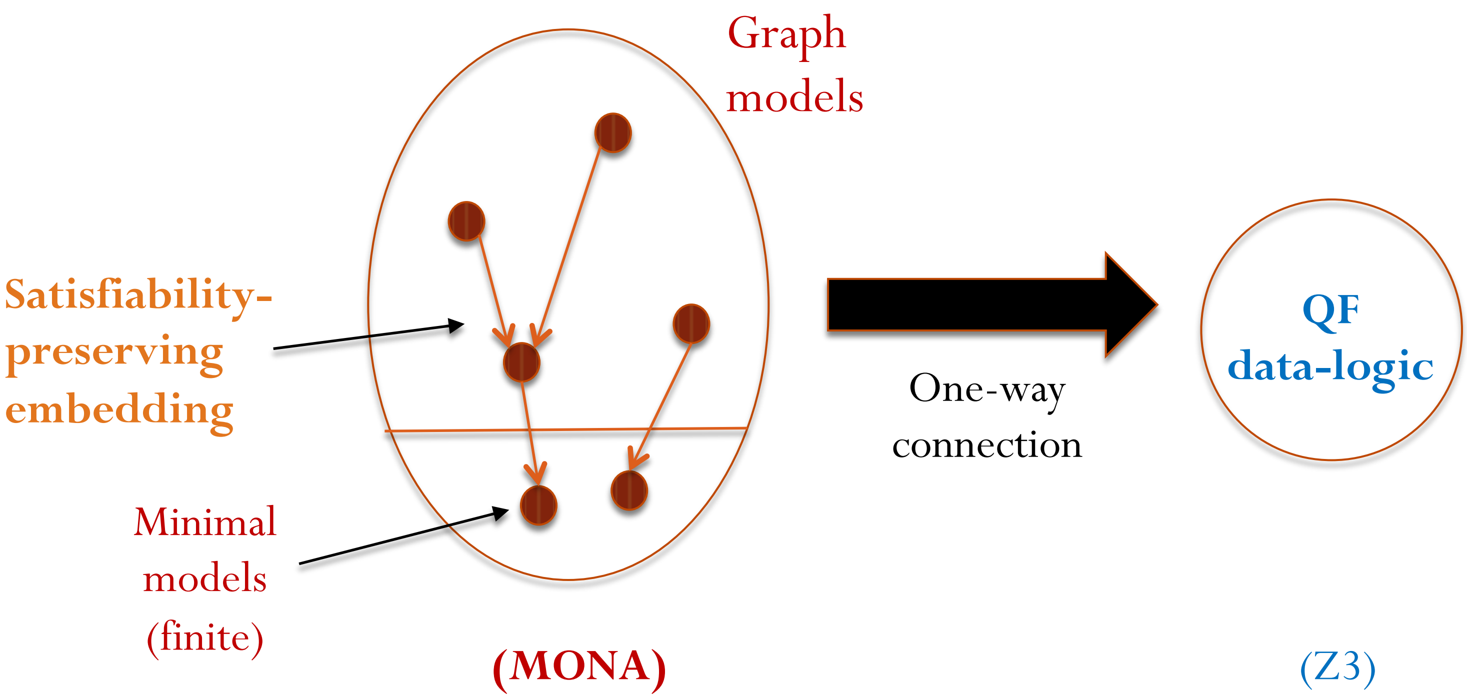
Decidable Fragments

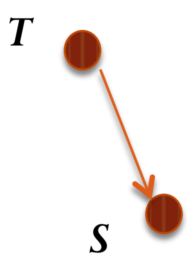
- Identify a semantic decidable fragment $STRAND_{dec}^{sem}$
  - semantically defined, but syntactically checkable
- Identify a syntactic decidable fragment $STRAND_{dec}$
- Use minimal models according to a *data-agnostic* embedding relation

Outline

- Semantic Decidable Fragment
- Syntactic Decidable Fragment
- Program Verification
- Experimental Results

Our Scheme

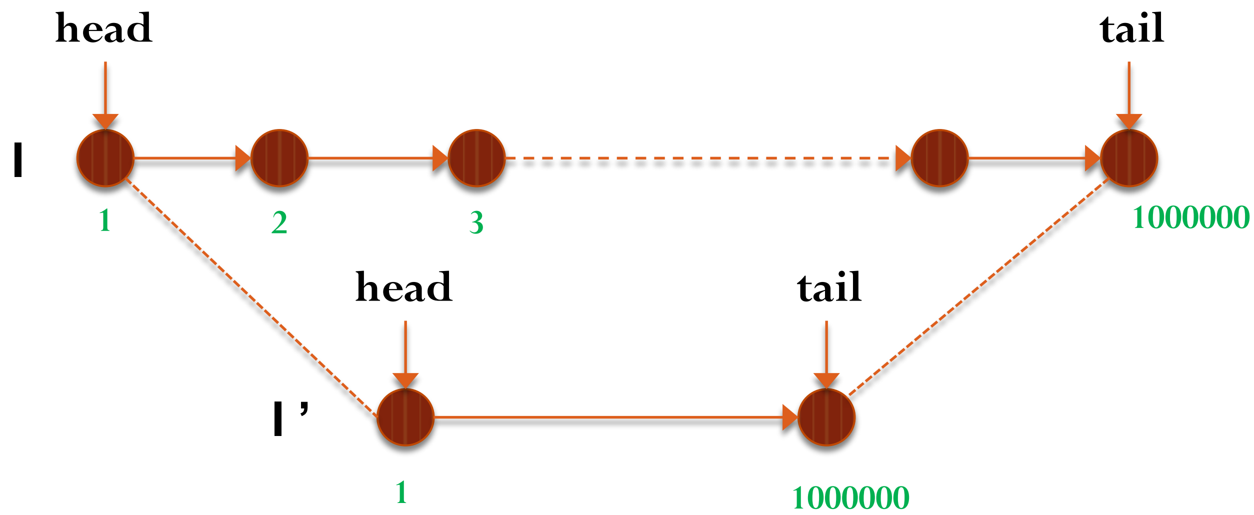




If *T* has a data-extension that satisfies the STRAND formula, then *S* also does.

Data-logic-agnostic!

Sorted List Embedding



$$\varphi_1 \equiv d(head) = 1 \wedge d(tail) = 10^6 \wedge \forall y_1 \forall y_2 ((y_1 \rightarrow^* y_2) \Rightarrow d(y_1) \leq d(y_2))$$

$$\hat{\varphi}_1 \equiv p_1(head) \wedge p_2(tail) \wedge \forall y_1 \forall y_2 ((y_1 \rightarrow^* y_2) \Rightarrow p_3(y_1, y_2))$$

Satisfiability-Preserving Embeddings

- S *satisfiability-preservingly embeds* in T iff

  no matter how T satisfies the formula using some valuation of the atomic data-relations,

  S will be able to satisfy the formula using the same valuation of the atomic data-relations

- T is a *minimal model* iff there is no proper submodel S of T such that S satisfiability-preserverly embeds in T

- $STRAND_{dec}^{sem}$: formulas that have finite number of minimal models w.r.t. the partial-order defined by satisfiability-preserving embeddings.

Observation

- Consider: $\psi = \exists\vec{x}\forall\vec{y}.\varphi(\vec{x},\vec{y})$

(Let $\gamma_1,\gamma_2,\ldots,\gamma_r$ be the atomic relational formulas of the data-logic in φ)

- Let M be a model of ψ , After fixing a particular valuation of $\vec{x}$ and $\vec{y}$, all data-relations γ_i get all fixed to true or false

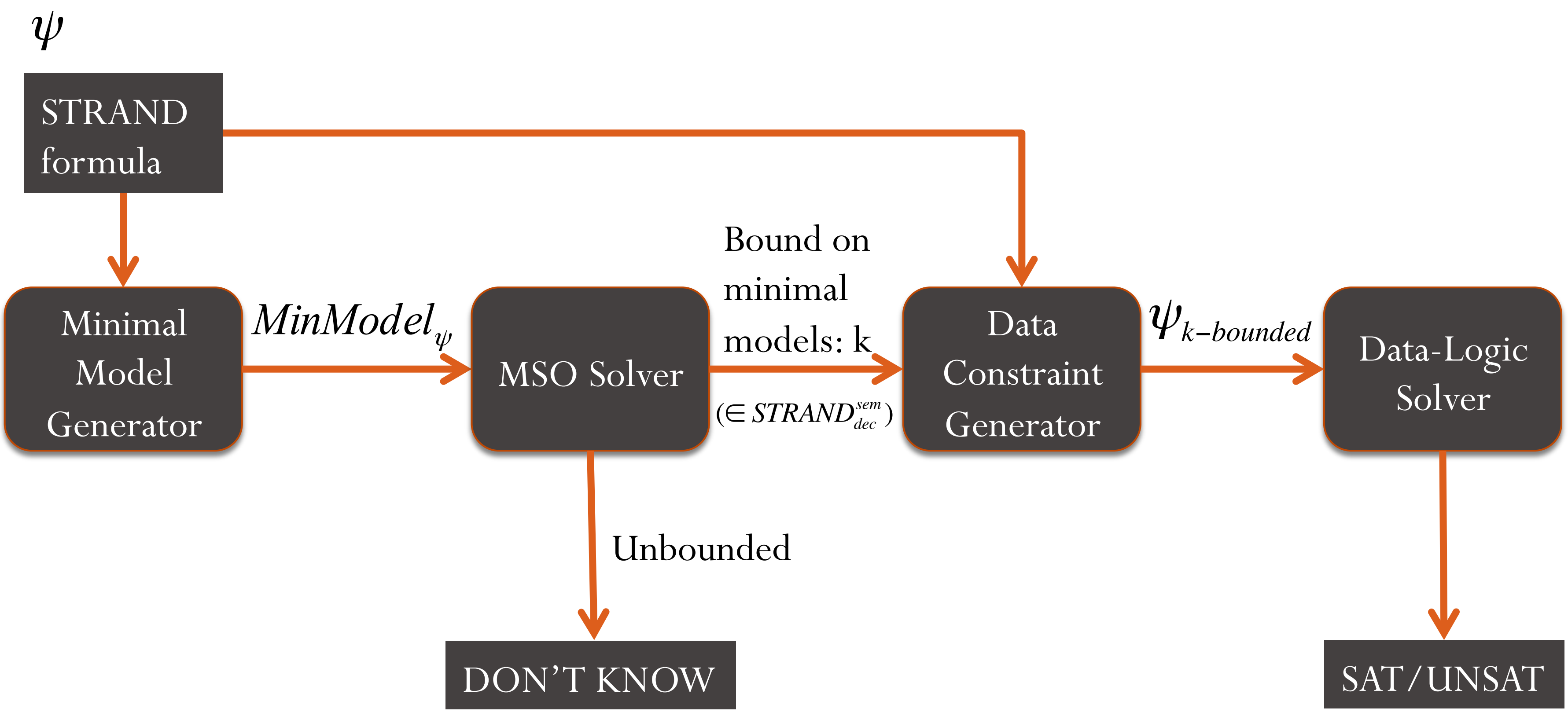
- Solution: From $\varphi(\vec{x},\vec{y})$, abstract γ_i as a predicate b_i to get a pure structural formula $\hat{\varphi}(\vec{x},\vec{y},\vec{b})$!

Minimal Model in MSO

$$
\begin{aligned}
MinModel_{\psi} \equiv \exists \vec{x}\,.\,(\neg \exists S\,.\,(\; & \\
& \vec{x} \subseteq S \wedge ValidSubmodel(S) \wedge \\
& \forall \vec{y} \in S, \vec{b}\,.\,(\\
& \quad \hat{\varphi}(\vec{x}, \vec{y}, \vec{b}) \\
& \quad\quad \Rightarrow Submodel(S)[\hat{\varphi}(\vec{x}, \vec{y}, \vec{b})])) \\
&) \\
)
\end{aligned}
$$

- transform the MSO formula to a tree automaton accepts precisely those minimal models
- Since the finite-ness of the language accepted by a tree automaton is decidable, $STRAND^{sem}_{dec}$ is effectively checkable!

Decision Procedure



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A syntactic decidable fragment

$STRAND_{dec}$: $\exists\vec{x}\forall\vec{y}.\varphi(\vec{x},\vec{y})$ where

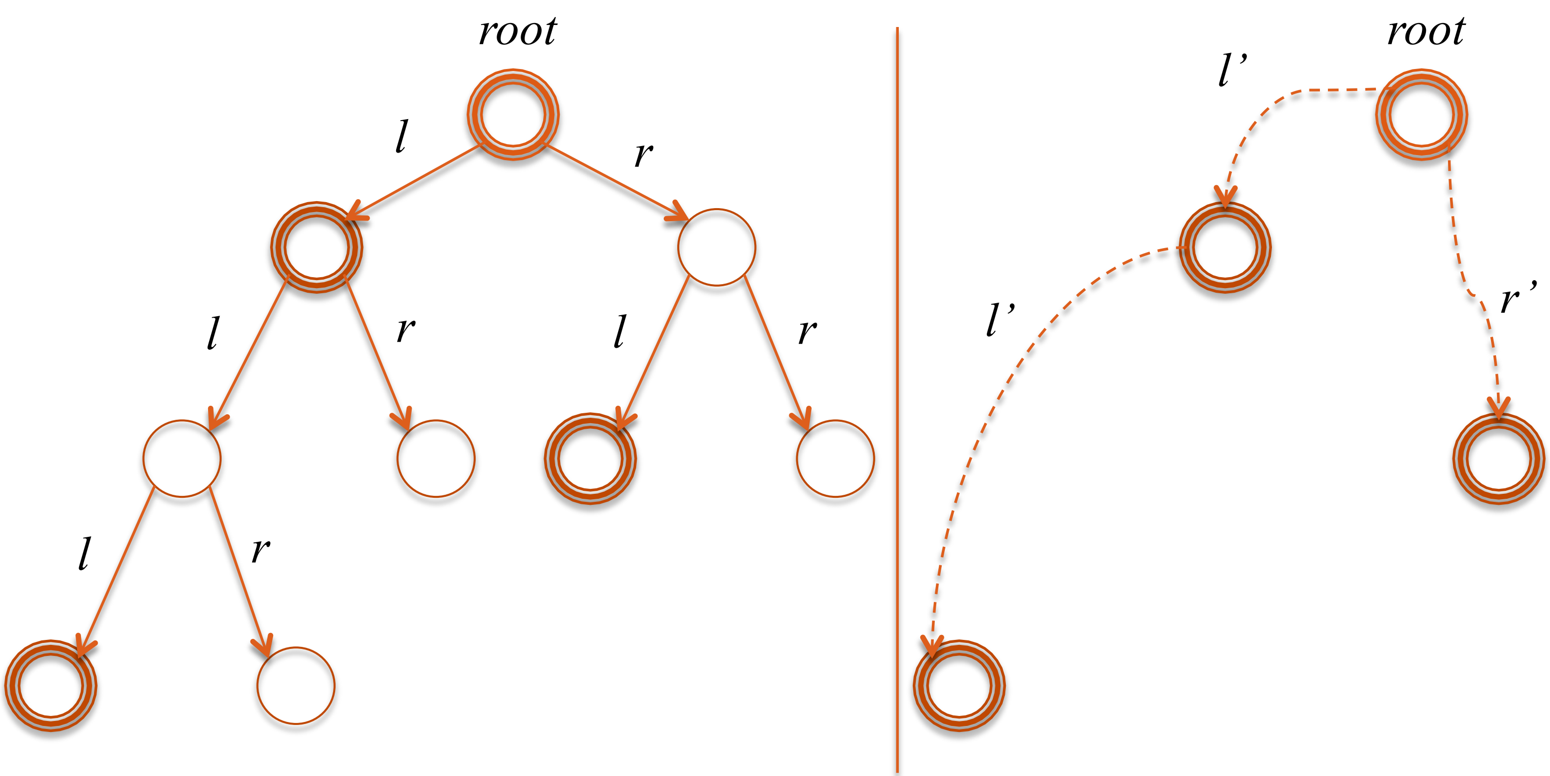
- φ has no quantification
- Elastic relations E and non-elastic relation NE
- Elastic relations are unconstrained
- Non-elastic relations only on $\vec{x}$ $NE(x_1,x_2)$

Elastic relations:

- $E(x,y)$ holds is model $\Leftrightarrow$ $E(x,y)$ holds in any submodel

$STRAND_{dec}$ is decidable!

Elastic Relations



A relation R is elastic if for any model M and its submodel M', for any $R(e_1, e_2)$, $e_1, e_2 \in M'$ holds for M iff $R(e_1, e_2)$ holds for M'.

E.g., *LeftSubtree(x, y)* is elastic but *LeftChild(x, y)* is not.

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Verification Conditions

Hoare-triples: (R, Pre, P, $Post$)
R: recursively defined data-structure

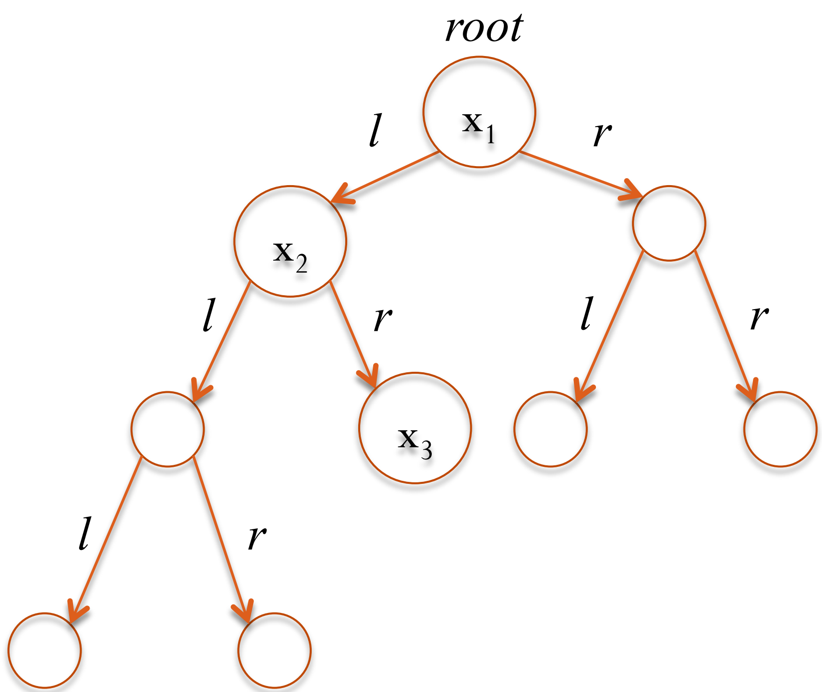
$Pre \in STRAND_{\exists,\forall}$
P:

```
new newhead ;
newhead.data := 14;
newhead := head ;
head := head.next ;
newhead.next := t ;
```

$Post \in STRAND_{\exists,\forall}$

Is $\psi \in STRAND$ satisfiable over R_P ?

Idea: capture the entire computation P starting from a particular recursively defined data-structure R using a single data structure R_P



$$Error \equiv \bigvee_{i\in[m]}\left(Pre_{R_P} \wedge \left(\bigwedge_{j\in[i-1]}\varphi_j\right) \wedge error_i\right)$$

$$Violate_{Post} \equiv Pre_{R_P} \wedge \left(\bigwedge_{j\in[m]}\varphi_j\right) \wedge \neg Post_{R_P}$$

Theorem

The Hoare-triple (*R, Pre, P, Post*) does not hold iff the STRAND formula $Error \vee Violate_{Post}$ is satisfiable on the trail R_P .

Program	Verification condition	Structural Solving (MONA)			Data-constraint Solving (Z3 with QF-LIA)		
		in $STRAND_{dec}^{sem}$? (finitely-many minimal models)	Time (s)	Graph model exists?	Bound (#Nodes)	Satisfiable?	Time (s)
Sorted-list-search	before-loop	Yes	0.34	No	-	-	-
	in-loop	Yes	0.59	No	-	-	-
	after-loop	Yes	0.18	No	-	-	-
Sorted-list-insert	before-head	Yes	1.66	Yes	5	No	0.02
	before-loop	Yes	0.38	No	-	-	-
	in-loop	Yes	4.46	No	-	-	-
	after-loop	Yes	13.93	Yes	7	No	0.02
Sorted-list-insert-error	before-loop	Yes	0.34	Yes	7	Yes	0.02
Sorted-list-reverse	before-loop	Yes	0.24	No	-	-	-
	in-loop	Yes	2.79	No	-	-	-
	after-loop	Yes	0.35	No	-	-	-
bubblesort	loop-if-if	Yes	7.70	No	-	-	-
	loop-if-else	Yes	6.83	No	-	-	-
	loop-else	Yes	2.73	Yes	8	No	0.02
bst-search	before-loop	Yes	5.03	No	-	-	-
	in-loop	Yes	32.80	Yes	9	No	0.02
	after-loop	Yes	3.27	No	-	-	-
bst-insert	before-loop	Yes	1.34	No	-	-	-
	in-loop	Yes	8.84	No	-	-	-
	after-loop	Yes	1.76	No	-	-	-
left-rotate	bst-preserving	Yes	1.59	Yes	19	No	0.05

http://cs.uiuc.edu/~qiu2/strand/

Conclusion

STRAND:

- A fundamental theory of combining decision procedures for graphs and data
- A semantic decidable fragment and a syntactic decidable fragment, both powerful enough for program verification
- Experimental results combining MONA and Z3 for analyzing heap programs.

Future Work:

- A fully-fledged engineering of an SMT solver for $STRAND_{dec}$
- An efficient non-automata theoretical decision procedure (unlike MONA) may yield more efficient decision procedures
- Decidable fragments of separation logic *with data*

Thank you!